

On Group Distance Magic Labeling of Circulant Graphs and Graph Products

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Abstract

Let $G = (V, E)$ be a graph of order n and let $D \subset \{0, 1, 2, 3, \dots\}$, for $v \in V$, the set $N_D(v) = \{u \in V : d(u, v) \in D\}$ is called the D -neighbourhood of v . A bijection $f : V(G) \rightarrow \{1, 2, \dots, n\}$ is called a D -vertex magic labeling of G if $\sum_{u \in N_D(v)} f(u) = k$ (a constant), for all $v \in V$. The constant k is called the D -vertex magic constant and a graph which admits a D -vertex magic labeling is called a D -vertex magic graph. Let $G = (V, E)$ be a graph of order n and let $v \in V$. The concept of D -vertex magic labeling has been extended to Abelian Groups as follows: For an abelian group Γ , a graph $G = (V, E)$ of the same order and $D \subset \{0, 1, 2, 3, \dots\}$, a D -Group distance magic labeling or a (Γ, D) -distance magic labeling of G is a bijection $f : V \rightarrow \Gamma$ such that $\sum_{x \in N_D(v)} f(x) = \alpha \in \Gamma$ for every vertex $v \in V$. A graph G of order n is said to be D -Group distance magic if it is (Γ, D) -distance magic with respect to every abelian group Γ of order n . In this paper, we study D -group distance magic labeling for various classes of circulant graphs. We establish necessary and sufficient conditions for the existence of such labelings in these graphs, highlighting the role of their structural properties. We further investigate D -group distance magic labeling in the context of graph products, with particular emphasis on Cartesian products involving multipartite graphs, paths, and cycles.

Keywords: Distance magic labeling, group distance magic labeling.

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